The desirability of Pay-As-You-Go Pensions when Relative Consumption Matters and Returns Are Stochastic

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Abstract

We investigate how preferences for relative consumption affect the desirability of a pay-as-you-go system when both labor productivity and returns on financial markets are stochastic and possibly correlated. More precisely, we compare the case in which prospective pensioners only care for absolute consumption with the case in which they also care about their consumption relative to the young. By comparing preferences that show the same local relative risk aversion, we are able to identify a necessary and sufficient condition for the optimal size of pay-as-you-go to be larger when relative consumption matters.

Keywords: pay-as-you-go pensions, fully funded pensions, relative consumption, risk aversion, relativity.

JEL classification: H55.
1. Introduction

Conclusions on the relative desirability of pay-as-you-go (PAYG) and fully funded (FF) pension systems have been obtained under the assumption that individuals care for their absolute consumption only.\(^1\) However, a growing body of literature suggests that individuals also care for their consumption relative to others’ (see Frey and Stutzer, 2000, 2002; Luttmer, 2005; Clark, Frijters and Shields, 2008, and references therein). The implications of individuals’ preferences for relative consumption have been studied with regard to a wide range of economic situations,\(^2\) but very little is known about the consequences of such preferences for the desirability of pension systems.\(^3\)

In this paper we contribute to explore this issue by investigating how the presence of concerns for relative consumption can affect the desirability of a PAYG system—as opposed to a FF—from the point of view of prospective pensioners when both future labor productivity and future returns on financial markets are stochastic and possibly correlated. Our main result is the identification of a necessary and sufficient condition for the optimal PAYG size to be larger when relative consumption matters. Furthermore, we discuss in what circumstances the relative consumption hypothesis makes a reasonable case for a larger PAYG size—i.e. for our condition to be satisfied. The basic intuition is the following. When people care for their future relative position in terms of consumption, productivity growth affects the value of a given amount of pension through its effect on the consumption of the future generation of young. Under dynamic efficiency, which we will take as the reference case, this affects the optimal PAYG size in two opposing ways. On the one hand, the PAYG becomes more attractive because it insures the prospective pensioner against the risk of being outperformed by others’ consumption. Indeed, while with standard preferences the PAYG is useful only to the extent that it helps to hedge financial risks, when relative consumption matters the PAYG becomes attractive also because it helps to increase the correlation between the consumption of prospective pensioners and

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\(^1\) In a situation of dynamic efficiency the steady state return from a FF is higher than the steady state return from a PAYG system, reducing the scope of the latter. However, when returns to both PAYG and FF are stochastic and not perfectly correlated, one can diversify risks by having a mix of PAYG and FF. Indeed, in recent years a number of papers have emphasized the role of social security in providing risk diversification with respect to several sources of risk, including return on financial markets, demographic and productivity shocks. See, among others, Gordon and Varian (1988), Shiller (1999), Dutta, Kapur and Orszag (2000) Bohn (2001, 2004), Wagener (2003b), Matsen and Thøgersen (2004), Krueger and Kubler (2006), Ball and Mankiw (2007).


\(^3\) To best of our knowledge the only paper dealing with this issue is Knell (2008) which — differently from us — takes an ex-ante perspective and deals with stochasticity by means of simulations.
that of next generation of young people—i.e. it insures prospective pensioner against the risk of being left behind by the next generation. On the other hand, the presence of concerns for relative consumption makes the PAYG potentially less capable of hedging risk because relative consumption directly depends on productivity growth. Indeed, while under standard preferences the PAYG pension works as an hedging device because the risks associated with productivity growth do not \textit{per se} enter the utility of prospective pensioners, under concerns for relative consumption the utility of prospective pensioners is already affected by changes in productivity, so that the PAYG pension is less effective in hedging this risk. Which of the two effects turns out to be dominant determines whether the presence of concern for relative consumption increases or decreases the desirability of the PAYG—from the point of view of prospective pensioners.

In making our point, we propose a strategy for coping with an issue that naturally arises when, in a stochastic environment, one needs to compare two situations that are characterized by different preference systems.\footnote{In our particular model we compare the case where prospective pensioners have concerns for relative consumption to the standard case where they only care about absolute consumption. We note that the same issue arises if we compare two situations characterized by different degrees of concerns for relativity.} There is no obvious way to introduce concerns for relative consumption \textit{ceteris paribus}. Different preference systems give rise to different relative risk aversions of prospective pensioners. Since relative risk aversion affects the desirability of the PAYG as a risk diversification instrument, it becomes unclear how to interpret changes in the optimal PAYG size due to a change in preferences: are they due to the introduction of concerns for relativity or are they due to different risk aversions? As we see no straightforward solution to this problem, our strategy is to restrict the analysis to comparisons between systems of preferences that show the same local relative risk aversion. In particular, we focus on a system of preferences for relativity that gives rise to the same local risk aversion obtained under the standard preference system and its optimal PAYG size. Our approach fits well the point of view of a policy-maker who is fully informed about people's local risk aversion—which might be interpreted behaviorally—but is uncertain about whether people have concerns for relativity or not. More precisely, one could imagine a policy-maker that begins with a PAYG whose size is optimal when only absolute consumption matters, and then tries to figure out in what direction the PAYG size should be changed if observed behavior were instead generated by preferences showing concerns for relativity.

The paper is organized as follows. In section 2 we present a simple OLG model suited for our purposes. In section 3 we determine the conditions for the optimal PAYG when individuals care only about their absolute consumption. In section 4 we modify the model to allow individuals to care also for their relative consumption. In particular, we illustrate

\footnote{In our particular model we compare the case where prospective pensioners have concerns for relative consumption to the standard case where they only care about absolute consumption. We note that the same issue arises if we compare two situations characterized by different degrees of concerns for relativity.}
and prove a necessary and sufficient condition that characterizes the cases in which concern for relative consumption enhances the role of the PAYG. Finally, we comment on what circumstances make it more likely that such a condition is satisfied. Section 5 concludes the paper with some final remarks about the scope and relevance of our findings.

2. The model

We consider a succession of generations $t, t+1, \ldots$, each one represented by a risk averse individual living two periods (we will use the terms “individual” and “generation” interchangeably). By considering a single representative for each generation we disregard issues of intra-generational distribution and risk sharing. Each representative individual works when young, and lives on savings and social security when old. We indicate consumption of generation $t$ when young (in period $t$) as $c_{1,t}$, and consumption when old (in period $t+1$) as $c_{2,t}$. Individual income at $t$ is denoted by $w_t$ and is taxed at a flat rate $\tau_t$ to finance a PAYG pension system. In particular, $\tau_tw_t$ is transferred to period $t$ pensioner – i.e. to individual born in $t-1$.

In order to focus on what is optimal from the point of view of prospective pensioners, we disregard all issues about the optimal consumption and savings of individuals when young. More precisely, since we are interested in how resources devoted to old age are best divided between PAYG and FF—and not in the overall pensions/savings levels—we simplify our analysis by assuming that the young’s consumption is determined mechanically as a fixed proportion of wage: $c_{1,t} = \gamma w_t$. This simplification allows us to study the model analytically without affecting the basic message of the paper. Indeed, assuming that consumption is a fixed proportion of wage is similar to what has been done in other studies on social security systems (e.g. Matsen and Thøgersen, 2004; Ball and Mankiw, 2007), where it is assumed that consumption by the young is zero; in our case, however, zero consumption would not be a reasonable simplification, as we want to consider variations in the level of consumption by the young induced by changes in productivity.\footnote{We are aware that changes in productivity would bring about adjustments in the intertemporal allocation. However, such additional effects bear upon the level of consumption/saving—and hence on the optimal overall amount of pensions—so that they are of second order with respect to the direct effect of concern for relative consumption on the optimal mix of FF and PAYG.}

Letting $s_t$ denote the savings of the young individual at $t$, we have that $\gamma w_t = w_t(1 - \tau - s_t)$ or $s_t = 1 - \tau_t - \gamma$. One can regard this condition as a purely behavioral assumption in order to abstract from costs and benefits experienced when young.

We do not explicitly consider capital accumulation. Instead, we take the growth rate $g_t$ from period $t$ to period $t+1$ and the interest rate $\tau_t$ as exogenously given. This hypothesis corresponds to the case of a small open economy. We also assume that $g_t$ and
\( r_t \) are stochastic, that they are not perfectly correlated, and that their joint distribution is elliptically symmetric. Moreover, all stochastic variables are independently and identically distributed over time. Finally, to make things simpler, we disregard demographic risk by assuming that population does not change from one period to another, and growth is entirely explained by changes in productivity.

Two last remarks on the nature of our approach are worth making at this stage, respectively about alternative perspectives on uncertainty and about the inter-generational contracts implied by different pension systems.

When looking at the desirability of a pension scheme, it is possible to consider two different approaches to risk. In the first place, we can take the point of view of each generation before its income in the first period is known, i.e. we might be interested in an ex ante perspective. Alternatively, we can take the point of view of the individual worker who already knows the realization of variables in their first period of life, i.e. we might be interested in the attitude of the active population toward social securities. We follow the latter approach as it is more consistent with our interest in the point of view of active population/prospective pensioners. The young individual at time \( t \) knows the realization of \( w_t \) but considers \( r_t \) and \( g_t \) and, hence, \( w_{t+1} = (1 + g_t)w_t \) as random variables. This is the so called interim perspective.

The properties of a PAYG system as an insurance device depend on the nature of the contract among generation because this affects the way the system adjusts to respond to shocks. In one case the pension system might grant each generation of pensioners a given level of benefits, defined as a function of their earnings in the previous period, and \( \tau_t \) is adjusted in each period to secure that the budget is balanced. Alternatively, the contribution rate \( \tau \) can be fixed for all generation, and the level of benefits is adjusted to satisfy the budget constraint. Under such a provision, each generation transfers a share \( \tau \) of its income to the previous generation, in exchange for a similar commitment by the next generation. It is clear that under the latter system (but not under the former) the effects of shocks affecting next period wage base can be shared between the next generation of workers and pensioners. Indeed, Hassler and Lindbeck (1998) show that, under the interim perspective on risk sharing (which they refer to as “true risk sharing”), it is only when \( \tau \) is fixed across generations that intergenerational risk sharing is possible and the pension system can provide hedging against the risky returns of private savings. For this

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6 The latter property will allow us to pass to a two-moment representation of preferences (Chamberlain, 1983; Eichner and Wagener, 2003). It requires that the level curves of the joint density function are all ellipses obtainable by means of an affine transformation of circumferences. Joint normality is a special case of elliptical symmetry.

7 Matsen and Thøgersen (2004) refer to the ex ante perspective as “Rawlsian” risk sharing.

8 For an extensive discussion of possible contracts and their effects, see e.g. Musgrave (1981).
reason we will focus on the case of $\tau = \tau$, disregarding other possible intergenerational contracts.\footnote{A possible interesting alternative from our point of view is a pension contract specifying that the ratio between the representative pension and the representative wage must be kept constant across periods. Note that this corresponds to the case of fixed $\tau$ under our hypothesis that population is stationary, while the two cases are not equivalent in general.} We note that, in this framework, $\tau \in [0, 1 - \gamma]$ can be interpreted directly as the chosen mix of FF and PAYG.

For the sake of comparison we begin our analysis assuming that individuals only care about their absolute level of consumption. Subsequently, we modify the model by positing that individuals also care about their consumption relative to a standard of living which depends on others’ consumption.

3. Case I: Only absolute consumption matters

For any given value of $\gamma$, consumption in the first period depends only on the realization of $w_t$. Therefore, the optimal choice of $\tau$ can be calculated considering only the expected utility of the second period, which we denote as $E[u(c_{2,t})]$. Second period consumption depends on the realization of both $r_t$ and $g_t$ and on the level of $\tau$:

$$c_{2,t} = s_t(1 + r_t)w_t + \tau(1 + g_t)w_t = [(1 - \gamma)(1 + r_t) + \tau(g_t - r_t)]w_t. \quad (1)$$

Since the joint distribution of $r_t$ and $g_t$ is elliptically symmetric, we can switch from $E[u(c_{2,t})]$ to a two-moment representation of preferences (Eichner and Wagener, 2003), and express the conditions identifying the optimal choice of $\tau$ for each generation as a function of the mean, variance and covariance of $g_t$ and $r_t$. This will turn out to be important for understanding the role played by the distributional parameters. More precisely, our representation will be based on the mean and the standard deviation of $c_{2,t}$:

$$E[c_{2,t}] = w_t[(1 - \gamma - \tau)(1 + \mu_r) + \tau(1 + \mu_g)] \quad (2)$$

$$S[c_{2,t}] = \sqrt{\text{Var}[c_{2,t}]} = w_t[(1 - \gamma - \tau)^2\sigma_r^2 + \tau^2\sigma_g^2 + 2\tau(1 - \gamma - \tau)\sigma_{rg}]^{1/2} \quad (3)$$

where $\sigma_r^2 = \text{Var}[r_t]$, $\sigma_g^2 = \text{Var}[g_t]$ and $\sigma_{rg} = \text{Cov}[r_t, g_t]$.

Preferences of individual $t$ are now described by the differentiable function

$$V(E[c_{2,t}], S[c_{2,t}]) \quad V_E > 0, V_S < 0 \quad (4)$$

where risk-aversion implies that $V$ is concave in its arguments (Meyer, 1987). By differ-
entiating (4) with respect to \( \tau \) we have that an increase in \( \tau \) increases utility whenever

\[
V_E \frac{\partial E[c_{2,t}]}{\partial \tau} + V_S \frac{\partial S[c_{2,t}]}{\partial \tau} > 0. \tag{5}
\]

The first order condition for an internal optimum, i.e. \( 0 < \tau^* < 1 - \gamma \), can be written as

\[
-\frac{V_S}{V_E} = \frac{(\mu_r - \mu_g)[(1 - \gamma - \tau)^2 \sigma_r^2 + \tau^2 \sigma_g^2 + 2\tau(1 - \gamma - \tau)\sigma_g \sigma_r]^{1/2}}{(1 - \gamma - \tau)(\sigma_r^2 - \rho_{gr} \sigma_r \sigma_g) - \tau(\sigma_g^2 - \sigma_r \sigma_g)} \tag{6}
\]

where the ratio \(-V_S/V_E\) is a measure of the local degree of risk aversion (Ormiston and Schlee, 2001; Lajeri and Nielsen, 2000; Wagener, 2003a).

Note that, in general, the optimal \( \tau \) depends on \( V_S/V_E \), which in turn may depend on the level of \( w_t \). Hence, the optimal \( \tau \) will be different for generations facing different realizations of \( w_t \). This problem of time inconsistency is well known in the literature on social security schemes (Lindbeck and Persson, 2003; Matsen and Thøgersen, 2004). In order to avoid it, we assume that preferences exhibit constant relative risk aversion.\(^{10}\)

Under this assumption, the ratio \( V_S/V_E \) is homogeneous of degree zero in its argument (see Meyer, 1987, Property 6), so that it does not depend on the realization of \( w_t \). This implies that it is possible to identify a single \( \tau^* \) which is optimal for all generations.

A pure FF system is optimal, i.e. \( \tau^* = 0 \), only if

\[
-\frac{\partial E[c_{2,t}]}{\partial \tau} = (\mu_r - \mu_g) > -\frac{V_S}{V_E} (\sigma_r - \rho_{gr} \sigma_g) = \frac{V_S}{V_E} \frac{\partial S[c_{2,t}]}{\partial \tau} \tag{7}
\]

where \( \rho_{gr} = \sigma_g/(\sigma_g \sigma_r) \) is the correlation coefficient between \( r_t \) and \( g_t \).

If returns on financial markets are, on average, lower than the growth of the contribution base, i.e. \( \mu_g > \mu_r \), then \( \tau^* > 0 \) independently of other parameter values. Instead, if \( \mu_g < \mu_r \), it is possible that \( \tau^* = 0 \). In particular, it is more likely that \( \tau^* = 0 \) the less individuals are risk averse, the more \( g_t \) and \( r_t \) are correlated and, provided that \( \rho_{gr} > 0 \), the lower is \( \sigma_r \) with respect to \( \sigma_g \) (Dutta et al., 2000; Matsen and Thøgersen, 2004).

In conclusion, if prospective pensioners have preferences for absolute consumption only and returns on financial markets are greater, on average, than the growth of the contribution base, then the role of a PAYG system is that of providing risk diversification. This is made clear by the fact that if financial investments are riskless, i.e. \( \sigma_r = 0 \), condition (7) is always satisfied and we have that \( \tau^* = 0 \).

\(^{10}\)This assumption, though quite strong, is consistent with the common observation that individuals usually show decreasing absolute risk aversion.
4. Case II: Relative consumption matters

We introduce concerns for relativity by positing that utility from consumption depends not only on absolute consumption but also on the current standard of living. The new utility function for the second period (old age) is \( u(c_{2,t}, \Sigma_{t+1}) \), where \( \Sigma_{t+1} \) is the standard of living at time \( t + 1 \), which we consider to be a linear function (convex combination) of \( c_{2,t} \) and \( c_{1,t+1} \). We further impose that a positive change in the standard of living can be translated into a negative change in absolute consumption according to a constant factor \( \delta > 0 \). In practice, the latter variable measures the intensity of concern for relative consumption. These two linearities imply that there exists a utility function \( \sim u \) such that

\[
\sim u(c_{2,t}, \delta c_{1,t+1}) = u(c_{2,t}, \Sigma_{t+1}),
\]

where \( \delta \in (0, \delta] \) represents how much the old generation values—in terms of its own consumption—an increase in the consumption of the young generation at \( t + 1 \).

By expressing the argument of \( \sim u \) as a function of \( g \) and \( r \) we see that \( c_{2,t} - \delta c_{1,t+1} \) is a linear combination of stochastic variables whose joint distribution is elliptically symmetric:

\[
c_{2,t} - \delta c_{1,t+1} = [(\tau - \beta \gamma)(1 + g) + (1 - \tau - \gamma)(1 + r)]w_t. \tag{8}
\]

Therefore, there exists the following two-moments representation of preferences over uncertain outcomes

\[
\tilde{V}(E[c_{2,t} - \delta c_{1,t+1}], S[c_{2,t} - \delta c_{1,t+1}]), \tag{9}
\]

where \( \tilde{V} \) is assumed to satisfy the same properties of \( V \) and where

\[
E[c_{2,t} - \delta c_{1,t+1}] = w_t[(1 - \gamma - \tau)(1 + \mu_r) + (1 - \tau - \gamma)(1 + \mu_g)] \tag{10}
\]

\[
S[c_{2,t} - \delta c_{1,t+1}] = w_t[(1 - \gamma - \tau)^2 \sigma_r^2 + (\tau - \beta \gamma)^2 \sigma_g^2 + 2(\tau - \beta \gamma)(1 - \gamma - \tau)\sigma_{rg}]^{1/2}. \tag{11}
\]

For the case where relative consumption matters, the counterpart of first order condition (6) is:

\[
-\frac{\tilde{V}_S}{\tilde{V}_E} = (\mu_r - \mu_g) \left[ (1 - \gamma - \tau)^2 \sigma_r^2 + (\tau - \beta \gamma)^2 \sigma_g^2 + 2(\tau - \beta \gamma)(1 - \gamma - \tau)\sigma_{rg} \right]^{1/2} \left( (1 - \gamma - \tau)(\sigma_r^2 - \sigma_{gr}) - (\tau - \beta \gamma)(\sigma_g^2 - \sigma_{gr}) \right). \tag{12}
\]

By letting individuals care about the standard of living \( \Sigma_t \), we have changed their preference system with respect to the case analyzed in the previous section. As discussed in the introduction, a change in the preference system can also change the degree of risk aversion. Since the desirability of a PAYG system as a risk diversification device is in part determined by individuals’ risk aversion, it becomes difficult to establish whether a
change in the optimal PAYG size is due to relativity concerns per se or, instead, it is an artifact of the change in risk aversion.

In order to overcome this problem we limit our analysis to the case where, at $\tau^*$, $V$ and $\tilde{V}$ give rise to the same attitude towards risk, i.e. we restrict to the class of utility functions such that $V_S/V_E = \tilde{V}_S/\tilde{V}_E$. In other words, we require that at the state of the world defined by $\tau^*$ the way in which individuals trade-off a small change in the mean and standard deviation of her consumption—when everything else is left unchanged—is independent of whether the individual cares about others’ consumption or not. We can frame such a restriction as a situation where the policy-maker is fully informed about people’s local risk aversion—which might be interpreted behaviorally—but is uncertain about whether people have concerns for relativity or not. More precisely, one could think as if the policy-maker begins with the optimal $\tau^*$ for the case where only absolute consumption matters and then tries to figure out in what direction the PAYG size should be changed if observed behavior were instead generated by preferences showing concerns for relativity.

Let $\tilde{\tau}^*$ denote the optimal contribution to the PAYG system when individuals have concerns for relativity. The following result is preliminary to our main statement:

**Lemma 1.** Let $0 < \tau^* < 1 - \gamma$. Under the assumptions that (i) $\mu_r > \mu_g$ and (ii) $V_S/V_E = \tilde{V}_S/\tilde{V}_E$ at $\tau^*$,

$$\tilde{\tau}^* > \tau^* \iff -\frac{\partial S[c_{2,t} - \beta c_{1,t+1}]}{\partial \tau} > -\frac{\partial S[c_{2,t}]}{\partial \tau}$$

(13)

**Proof.** From the fact that $V$ is concave in $(E,S)$, $E$ is linear and $S$ is strictly convex at $\tau$, follows that $V$ is strictly concave in $\tau$. The same is true of $\tilde{V}$.

To simplify notation, we write $E'$ for $\partial E[c_{2,t}]/\partial \tau$ and $S'$ for $\partial S[c_{2,t}]/\partial \tau$; similarly, let $\tilde{E}'$ be $\partial \tilde{E}[c_{2,t} - \beta c_{1,t+1}]/\partial \tau$ and $\tilde{S}' = \partial \tilde{S}[c_{2,t} - \beta c_{1,t+1}]/\partial \tau$.

Assume that $0 < \tau^* < 1 - \gamma$. Strict concavity implies that $\tilde{\tau}^* > \tau^*$ if and only if $\tilde{V}$ is increasing in $\tau$ at $\tau^*$. The first order conditions for a maximum imply (i). From (ii) and from the fact that $E' = \tilde{E}'$ follows that $\tilde{V}$ is increasing at $\tau^*$ if and only if $\tilde{S}' < S'$. □

Figure 1 helps to get the intuition behind Lemma (1). It shows the optimality conditions in the $(S,E)$-space when $0 < \tau^* < (1 - \gamma)$ and $\partial S[c_{2,t} - \beta c_{1,t+1}]/\partial \tau < \partial S[c_{2,t}]/\partial \tau$.

Suppose first that only absolute consumption matters. If $\mu_r > \mu_g$, then the locus of feasible combinations of standard errors and expected values determined by $\tau$ is given by the curve $CC$ (where $E$ decreases in $\tau$). The slope of the $CC$ curve is equal to the

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11 It can be easily checked that the second order derivative of $S$ with respect to $\tau$ is positive—hence strict convexity is granted—provided that $\tau_r$ and $g_t$ are not perfectly correlated.
Figure 1: Unlabeled curves represent indifference curves in the \((S, E)\)-space. Curve \(CC\) identifies the locus of feasible combinations of standard errors and expected values as induced by \(\tau\) under concerns for absolute consumption only. Curve \(C'C'\) does the same under concerns for relative consumption. Since \(\mu_r > \mu_s\), a greater \(\tau\) induces a lower \(E\). The optimal \(\tau\) is obtained when the curve is tangent to the highest feasible indifference curve.

ratio between the derivatives of \(E\) and \(S\) with respect to \(\tau\), and the shape of the curve reflects the fact that \(E\) is linear in \(\tau\) while \(S\) is convex in \(\tau\). Concavity of \(V\) implies that indifference curves are convex. The condition that identifies \(\tau^*\) is the tangency between the \(CC\) curve and the highest feasible indifference curve.

Consider the change in preferences induced by the introduction of concerns for relativity, which is represented by the passage from \(u\) to \(\bar{u}\). As explained above, we restrict the analysis to new preference systems characterized by an unchanged attitude towards risk at \(\tau^*\). More precisely, since relative risk aversion does not change at \(\tau^*\), the slope of the indifference curve is unchanged at that point. \(^{12}\) Hence, a necessary and sufficient condition for \(\bar{\tau}^* > \tau^*\) is that the slope of the curve representing the new locus of feasible combinations of standard errors and expected values, namely \(C'C'\), is lower at \(\tau^*\). This is what lemma 1 establishes. \(^{13}\)

By developing inequality (13) we obtain an explicit necessary and sufficient condition for \(\bar{\tau}^* > \tau^*\) based on the model parameters:

**Proposition 1.** Under the assumptions that (i) \(\mu_r > \mu_s\) and (ii) \(V_S/V_E = \bar{V}_S/\bar{V}_E\) at \(\tau^*\):

\(^{12}\) Relative risk aversion is defined, under the new preferences, with reference to consumption adjusted for concern for relative consumption, i.e. with respect to \(c_{i,t} - \beta c_{i,t-1}\). Of course, this does not imply constant relative risk aversion with respect to changes in \(c_{i,t}\) alone.

\(^{13}\) Note that, in order to make the graph of \(C'C'\) comparable with that of \(CC\), we have rescaled both horizontal and vertical axis in such a way that \(E[c_i] = E[c_i - \beta c_i]\) and \(S[c_i] = S[c_i - \beta c_i]\). This is by no means with a loss of generality.
(a) If $0 < \tau^* < 1 - \gamma$, then $\tilde{\tau}^* > \tau^*$ if and only if

$$\beta \left[ 1 - \frac{(1 - \gamma)(\sigma_g^2 - \rho_g\sigma_r)}{(1 - \gamma - \tau)(\sigma_g^2 - \rho_g\sigma_r) - \tau(\sigma_g^2 - \rho_g\sigma_r)} \right] < 2 \left( \frac{1 - \gamma}{\gamma} \right).$$

(a) If $\tau^* = 0$, then $\tilde{\tau}^* > 0$ only if (14); moreover, (14) implies that for any $\beta \in (0, \delta]$, there exists a high enough degree of risk aversion at $\tau^* = 0$ such that $\tilde{\tau}^* > 0$. □

PROOF. Part a). From lemma 1 we know that, under assumptions (i) and (ii), $\tilde{V}$ is increasing at $\tau^*$ if and only if

$$\tilde{S}' = \frac{-(1 - \gamma - \tau)(\sigma_r^2 - \sigma_g) + (\tau - \beta\gamma)(\sigma_g^2 - \sigma_g)}{[(1 - \gamma - \tau)^2 \sigma_r^2 + (\tau - \beta\gamma)^2 \sigma_g^2 + 2(\tau - \beta\gamma)(1 - \gamma - \tau)\sigma_g]} \left( 1 + \frac{(1 - \gamma - \tau)(\sigma_r^2 - \sigma_g) + \tau(\sigma_g^2 - \sigma_g)}{[(1 - \gamma - \tau)^2 \sigma_r^2 + \tau^2 \sigma_g^2 + 2\tau(1 - \gamma - \tau)\sigma_g]} \right)^{1/2} < \tilde{S}' = S' \quad (15)$$

from which straightforward calculations lead us to

$$\beta\gamma \left[ (1 - \gamma - \tau)(\sigma_r^2 - \sigma_g^2) - 2\tau(\sigma_g^2 - \sigma_g) \right] < 2(1 - \gamma)(1 - \gamma - \tau)(\sigma_r^2 - \sigma_g) - \tau(\sigma_g^2 - \sigma_g)].$$

Note that the term in squared brackets on the right hand side must be positive at $\tau^*$, since the numerator of $S'$ must be negative at an interior optimum. We divide both sizes by this term and obtain (14).

Part b). Suppose that $\tau^* = 0$. The first order condition for the maximization of $V$ implies that at $\tau = 0$ we have $S' \geq -(V_{\beta} \tilde{E}')/V_S$. From the first derivative of $\tilde{V}$ with respect to $\tau$ we get that, at $\tau = 0$, $\tilde{V}$ increases in $\tau$ only if $S' < -(\tilde{V}_{\beta} \tilde{E}')/\tilde{V}_S$. Hypotheses (i)-(ii) imply that, at $\tau = 0$, $\tilde{V}$ increases in $\tau$ only if $\tilde{S}' < S'$. As shown above, the latter condition is equivalent to (14). Finally, form strict concavity follows that if $\tilde{S}' < -(\tilde{V}_{\beta} \tilde{E}')/\tilde{V}_S$ then $\tilde{\tau}^* > 0$. Since $\tilde{E}'$ is a constant, $\tilde{S}'$ is bounded from below, while $\tilde{V}_S/\tilde{V}_E$ is both negative and unbounded from below, we conclude that for every $\beta \in (0, \delta]$ there exists a degree of risk aversion at $\tau = 0$ such that $\tilde{S}' < -(\tilde{V}_{\beta} \tilde{E}')/\tilde{V}_S$.

Proposition 1 states under what circumstances the presence of concern for relative consumption either enhances or diminishes the scope of a PAYG system. In order to give a better intuition of this result, it is useful to decompose the effect of a marginal increase in $\tau$ on $S[c_{2,t} - \beta c_{1,t+1}]$, evaluated at $\tau^*$, into two distinct effects: the first due to a reduction in $S[c_{2,t}]$, the second due to an increase in the correlation between $c_{2,t}$ and $c_{1,t+1}$.  

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Condition (13) can be rewritten as:

\[
- \frac{\partial S[c_{2,t}]}{\partial \tau} \cdot \frac{S[c_{2,t}] - \rho_{12} \beta S[c_{1,t+1}]}{S[c_{2,t} - \beta c_{1,t+1}]} + \frac{\partial \rho_{12}}{\partial \tau} \cdot \frac{\beta S[c_{2,t}] S[c_{1,t+1}]}{S[c_{2,t} - \beta c_{1,t+1}]} > - \frac{\partial S[c_{2,t}]}{\partial \tau} \tag{17}
\]

where \( \rho_{12} \) denotes the correlation coefficient between \( c_{2,t} \) and \( c_{1,t+1} \).

The reason why concerns for relative consumption may entail \( \tilde{\tau}^* > \tau^* \) is that they make it more attractive to tighten the link between changes in \( c_{2,t} \) and changes in \( c_{1,t+1} \), which of course can be accomplished by increasing \( \tau \). This effect is captured by the second term of (17) which is always positive and hence makes the marginal effect of \( \tau \) on \( S[c_{2,t} - \beta c_{1,t+1}] \) larger with respect to the case where people care about absolute consumption only.

However, it must also be considered that when relative consumption matters a reduction in \( S[c_{2,t}] \) is important only inasmuch as it results in a reduction in \( S[c_{2,t} - \beta c_{1,t+1}] \). The actual effect on \( S[c_{2,t} - \beta c_{1,t+1}] \) depends on aspects like the overall volatility of consumption and the correlation between the consumption of the old and the young. Since the fraction in the first term of (17) is never higher than one, the effect of a greater \( \tau \) on \( S[c_{2,t} - \beta c_{1,t+1}] \) through a change in \( S[c_{2,t}] \) is not larger than the change in \( S[c_{2,t}] \) itself. In other words, the volatility of the random variable \( c_{2,t} - \beta c_{1,t+1} \) is less sensitive to \( \tau \) than the volatility of the random variable \( c_{2,t} \). This is because \( c_{2,t} - \beta c_{1,t+1} \) already bears a risk associated with productivity growth so that an increase in \( \tau \) is less effective in hedging risk than in the case where utility when old only depends on \( c_{2,t} \).

Thus, the two effects should be balanced one against the other. Whenever the benefit from a reduction in \( S[c_{2,t}] \) offsets the benefits from an increase in \( \rho_{12} \) the scope of PAYG diminishes, otherwise it increases – i.e. condition (14) is satisfied.

From (17) we see that much depends on both sign and size of \( S[c_{2,t}] - \rho_{12} \beta S[c_{1,t+1}] \). In particular, if \( \rho_{12} \) is so large that \( S[c_{2,t}] - \rho_{12} \beta S[c_{1,t+1}] \) is bad from the point of view of the risk-averse prospective pensioner because \( S[c_{2,t} - \beta c_{1,t+1}] \) increases. A large \( \rho_{12} \) is the effect of large \( \rho_{gr} \) and \( \tau \): in these circumstances, the link between \( c_{2,t} \) and \( c_{1,t+1} \) cannot be increased much by increasing \( \tau \), and hence the FF may turn out to be more attractive than the PAYG. This however is possible only if \( \beta \) or \( \gamma \) are large enough as \( \rho_{12} \) cannot be greater than unity. In particular, \( \beta \) and \( \gamma \) crucially affects the relative magnitude of \( c_{2,t} \) and \( c_{1,t+1} \), and hence that of \( S[c_{2,t}] \) and \( S[c_{1,t+1}] \).

The described circumstances are logically feasible, but not necessarily economically relevant. The following sufficient conditions for \( \tilde{\tau}^* > \tau^* \) shed some light on their economic

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\[ (S[c_{i,t}] - \rho_{i,t} \beta S[c_{i,t+1}])^2 \leq \frac{S[c_{i,t}] - \rho_{i,t} \beta S[c_{i,t+1}]}{S[c_{i,t}] - \beta c_{i,t+1}} \quad \text{where the equality is only for } |\rho_{i,t}| = 1. \]
relevance. Since the term in squared brackets in (14) cannot be larger than 2 for \( \tau \leq (1-\gamma) \), and it is certainly not larger than unity when \( \sigma_g^2 - \rho_{gr}\sigma_g\sigma_r \geq 0 \), we obtain:

**Corollary 1.** If \( 0 < \tau^* < 1 - \gamma \), then \( 0 < \beta \leq (1 - \gamma)/\gamma \) implies \( \tilde{\tau}^* > \tau^* \). Moreover, if \( \rho_{gr} \leq \sigma_g/\sigma_r \), then \( 0 < \beta \leq 2(1 - \gamma)/\gamma \) implies \( \tilde{\tau}^* > \tau^* \).

In our opinion, there are some good reasons to believe that either of these sufficient conditions will often be satisfied. First of all, a non-negligible part of a pensioner’s reference group is presumably made of other pensioners. Hence, the relevant standard of living will depend only in part on the consumption of the young. This suggests that the parameter \( \beta \), representing the sensitivity of the old with respect to the young’s consumption will be bounded away from \( \delta \leq 1 \).

Secondly, it may be argued that only a part of the consumption of the young is used as a reference by the old. Consumption is partly age-specific, and the consumption of the young is usually different from that of the old. Moreover, a fraction of the consumption of the young can actually be an income production cost (e.g. baby sitting, commuting costs, etc.) and, hence, it should not be taken into account in the life standard. This argument too points to a \( \beta \) not very close to \( \delta \).

Lastly, there is the issue of how large is \( \gamma \) with respect to \( 1 - \gamma \). In a two period framework, consumption smoothing suggests that the desired amount of savings \( 1 - \gamma \) will not be very far from one-half, making \((1 - \gamma)/\gamma \) close to unity. The fact that we usually observe a lower saving rate, closer to one-third, is presumably related to the fact that in the life cycle the average working time is approximately twice as much as the time of retirement. To put it differently: when comparing total consumption during the working age with total consumption at old age, the different lengths of the two periods over which consumption is spread should be taken into account, and \( \beta \) scaled down accordingly.\(^{15}\)

Finally, it is worth emphasizing that when relative consumption matters the role played by the PAYG pension system is not simply that of risk differentiation in the face of financial markets volatility, as already emphasized in the literature. This becomes evident if we consider the hypothetical case in which \( r_t \) is deterministic. In this case, when only absolute consumption matters, the condition \( \mu_r > \mu_g \) is enough to secure that \( \tau^* = 0 \). On the contrary, when we consider concern for relative consumption, the optimal value of \( \tau \) can be positive even if \( \mu_r > \mu_g \), and the chance that \( \tilde{\tau}^* > 0 \) increases with \( \sigma_g^2 \). This is established by the following

\(^{15}\) Note that the inclusion of a positive demographic growth would reinforce this argument, since the same amount of resources would be spread over a number of pensioners which is lower than the number of workers.
Corollary 2. If \( r_t \) is non-stochastic, then \( \tilde{r}^* > 0 \) if and only if
\[
\frac{-V_E g}{\sigma_g} > \mu_r - \mu_g \tag{18}
\]

Proof. Under the assumption that \( \mu_r > \mu_g \), the fact that \( r_t \) is non-stochastic implies that \( \tilde{r}^* = 0 \). Therefore, the first order condition for a maximum of \( \tilde{V} \) reduces to (18).

5. Concluding remarks

In this paper we have investigated how concerns for relative consumption may affect the desirability of a PAYG pension scheme when both labor productivity and returns on financial markets are stochastic and possibly correlated. We have done this by adopting the point of view of prospective pensioners and comparing the desirability of PAYG under two different specifications of preferences: in the first prospective pensioners are concerned with their absolute consumption only, in the second they also care about their consumption relative to others.\(^\text{16}\)

One might think that concerns for relative consumption make a PAYG systematically more or less desirable. Our main result shows that this is not the case: the optimal PAYG size may either increase or decrease depending on the parameter values—i.e. the correlation between PAYG and FF returns, the relevance of young’s consumption in setting the standard of living, and the intensity of preferences for relativity. The intuition behind our result is the following. The fact that prospective pensioners care for their future relative position makes future productivity relevant to them. The reason is that future productivity affects the consumption of future generations of young, hence contributing to set the standard of living people compare with. This in turn affects the optimal PAYG size in two opposing ways. First, a growth in future productivity becomes a bad for prospective pensioners as it reduces pensioners’ relative consumption. Hence, the PAYG becomes more attractive due to its capability to insure the prospective pensioner against being outperformed by others’ consumption. Second, the PAYG becomes potentially less effective as an instrument of risk diversification. Indeed, since relative consumption already bears a risk associated with productivity growth, linking consumption in the old

\[^{16}\text{This analysis is related to the one developed independently by Knell (2008), who claims that the role of the PAYG system is enhanced when pensioners care for the consumption of the active generation, and provides a calibration-based quantitative assessment of such an effect. There are however some differences between the two papers: being interested in the point of view of prospective pensioners, we assume an interim rather than an ex ante perspective; moreover, we fully take into account the role of the correlation between the implicit returns on PAYG and FF by considering that both productivity and returns from financial markets are stochastic and may be correlated.}\]
age to productivity growth is less effective in hedging risks. Which of these two effects is dominant determines how relativity concerns affect the desirability of a PAYG scheme.

In our analysis we have also dealt with a difficulty that naturally arises in this setup. As different preference systems imply different risk aversions, it becomes difficult to isolate the effect of relativity concerns per se. We have coped with this problem by restricting the analysis to comparisons between systems of preferences that show the same local relative risk aversion. Intuitively, the situation that we assess is one where the policy-maker begins with the PAYG size which is optimal under the assumption that only absolute consumption matters and then tries to figure out in what direction the PAYG size should be changed if the observed behavior—i.e. local risk aversion—were instead generated by preferences showing concern for relative consumption.

A few final remarks on our main finding and its scope are worth considering.

In order to focus on our core issue, we have abstracted from various important aspects related to pension systems such as the effects on savings and on the labor market (see e.g. Lindbeck and Persson, 2003). Of course, the overall desirability of a PAYG system also depends on these issues. In this respect we emphasize that, in order to inform decision-makers, our findings need to be integrated in a more general analysis. Actually, we think that a careful exploration of this possibility should be the next step along this line of research.

Another simplifying assumption of our study is that of constant relative risk aversion. This was made to avoid time inconsistency in the optimal size of the PAYG system. Although we cannot dispense with such an assumption if we want to retain time consistency, we emphasize that this is not crucial for our main result. More precisely, the necessary and sufficient condition provided in our proposition 1 would still be true, though it should be checked for each period separately, as the optimal FF-PAYG mix could change over time.

We have also disregarded the role that may be played by demographic risk. As it is often emphasized, demographic risk may be very relevant in the design of the pension system and, in particular, in determining the optimal mix between PAYG and FF. However, we do not see compelling reasons for demographic risk to affect the consumption of the future young directly. Therefore, we think that demographic risk would not have first order effects on how concerns for relative consumption affect the desirability of pension schemes.

Finally, we have derived our results under the assumption that people care about relative consumption in a cardinal way—more precisely, that what matters is the difference between current consumption and the standard of living. As shown by Bilancini and Boncinelli (2008), this may be a non-innocuous assumption. A good robustness check
of our results would be to repeat the analysis under alternative specifications of concern for relative consumption. At any rate, since our necessary and sufficient condition for a greater PAYG is local—i.e. it considers the desirability of marginal changes in the PAYG size—it's validity is independent of the shape of relativity concerns because locally they can be always approximated in a linear way.
References


